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On the Probability of Flying Through Nuclear Dust Clouds

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Abstract

Since the Mount St. Helen's eruption, the dust clouds formed by a nuclear detonation have become of much more interest to the Air Force acquisition community. The dust from nuclear detonations can have a severe impact on aeronautical systems, such as engines, surface coatings, and wind screens. This paper presents an analytic approach to calculating the probability that an aircraft will encounter 0 to N dust clouds when flying through an area. Given the size of a rectangular area containing nuclear dust clouds, and the size and the number of the dust clouds, the probability of encounter can be easily calculated. This approach to calculating the probability of encounter is useful for system studies when the details of the nuclear attack that generates the dust clouds is not known. The method is suitable for implementation in a computer code or a spreadsheet.

On the Probability of Flying Through Nuclear Dust Clouds

Introduction

Since the Mount St. Helen's volcanic eruption, the effects of dust clouds from nuclear detonations have become of much more interest to the Air Force acquisition community. It is relatively easy to model the dust cloud environment from a single nuclear detonation. (See [1], [2], and [3], for instance.) It requires a few weeks to develop the model, and a few seconds to calculate the effects of flying through the cloud. It is not so easy to model the probability of flying through a cloud. (See [4], for instance.) The model must use a large selection of wind patterns to build a statistically significant database of cloud encounters, and the location of hundreds to thousands of dust clouds must be compared to the flight paths of hundreds of cruise missiles to calculate the probability of encounter, and the effects of the encounters. System studies early in the acquisition process, such as for an Analysis of Alternatives (AoA), may not have all of the information needed to perform a detailed analysis of the type described by Rausch in reference [4]. The nuclear lay downs that generate the dust clouds may not be available, and some details such as the yields and heights of burst may not have been determined. Also, it may be prohibitively expensive to run the dust cloud simulation codes multiple times to allow for a large enough trade space to meet the needs of the AoA. There is a need for a simple, fast running method to estimate the probability of flying through a nuclear dust cloud.

This paper presents a simple analytic method for calculating the probability of flying through a nuclear dust cloud. To use this method, the analyst must be able to estimate the size of a rectangular area that contains the dust clouds. The analyst also needs to know the number and size of the clouds. The analyst can then calculate the probability that an aircraft flying through the area will intercept at least one dust cloud, or the probability of intercepting some number of N dust clouds. The method is suitable for implementation in a computer code or a spreadsheet.

Model Development for Clouds All One Size

Figure 1 below shows a rectangular area with four circles with each circle representing a nuclear dust cloud. The dust clouds have a diameter D. The area has a length and width of L and W, respectively. The arrow shows the proposed flight path of an aircraft through the area. Assume that the dust clouds have been randomly placed. Further, assume that the flight path has been randomly picked, but that the direction of flight is constrained to be parallel to the side of the rectangle, as shown. We want to calculate the probability that the aircraft flies through 0, 1, 2, 3, or 4 clouds.

From Figure 1 it would appear that the probability would be calculated by comparing the area and random placement of the clouds to the area of the enclosing rectangle. However, it is possible to reduce the problem to one-dimension. Figure 2 shows the same cloud placement as Figure 1, but now the "shadow" cast by each cloud has been extended to the right side of the rectangle.

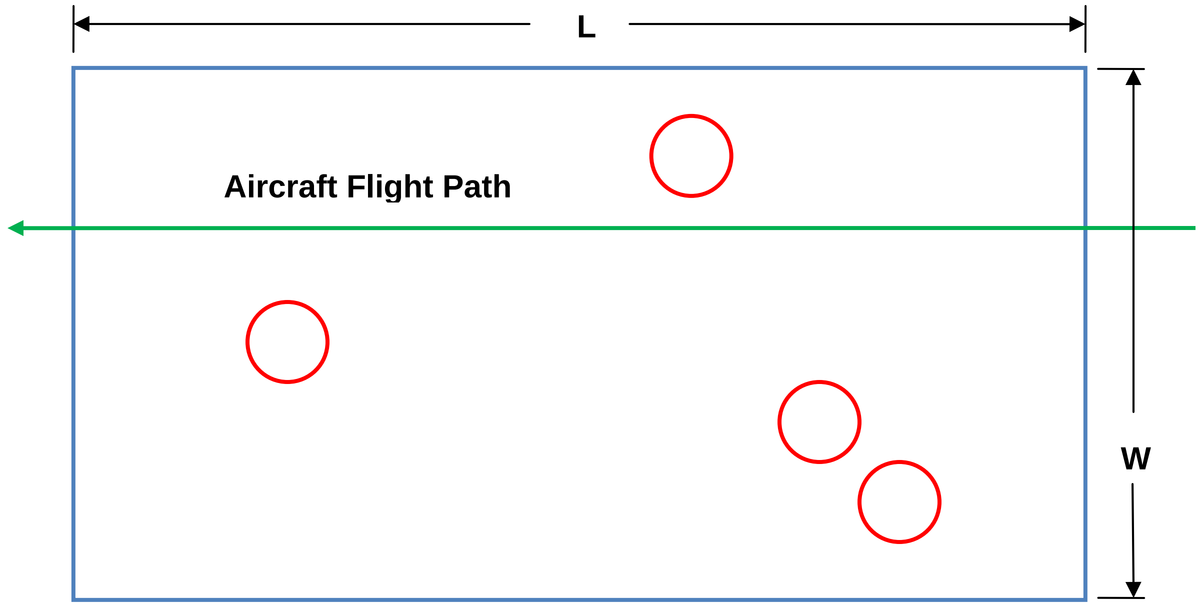

Figure 1-Clouds randomly located within an area

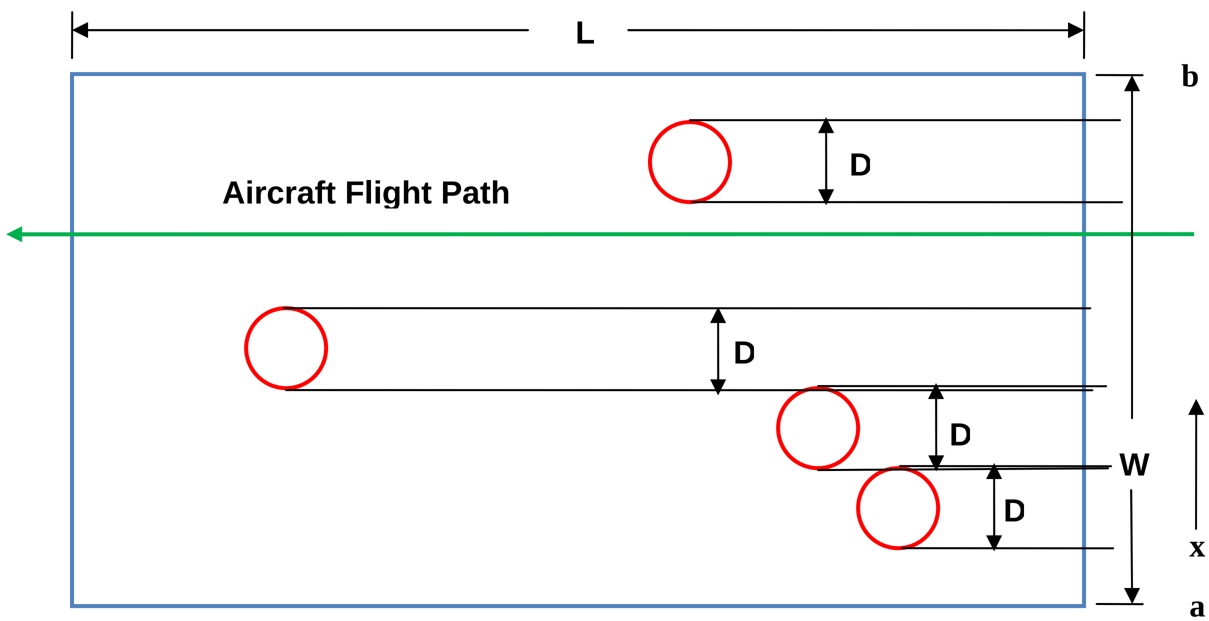

Figure 2-Shadows cast by randomly placed clouds within an area

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In Figure 2 the x axis extends upward, and the coordinates of the endpoints of the width of the rectangle are at the coordinates a and b. If the cloud locations are picked with a uniform random draw, then the probability that any point x along the width of the rectangle will be the location for the center of a cloud can be expressed with a uniform distribution:

$$f(x; a, b) = \frac{1}{b - a} \quad for\ a \le x \le b, \qquad 0\ otherwise \qquad (1)$$

Since b - a is simply the width, W, of the rectangle, and including the diameter, D, of a cloud, the probability that any point along x is captured within the interval D from a single cloud is:

$$f(x; a, b) = \frac{D}{W} \quad for\ a \le x \le b, \qquad 0\ otherwise \qquad (2)$$

Note that Equation (2) neglects edge effects. That is, if a point x is selected near either a or b, such that part of the interval D is less than a, or greater than b, then Equation 2 ignores that part of the D interval which is outside of the line width. This is acceptable if D << W, so that there is very little error in the probability in Equation (2). Alternatively the intervals can be assumed to "wrap around" the width. If part of an interval is outside of the width, it is "wrapped around", and added to the other side of the rectangle. This is the approach assumed in this paper, so that any D and W ratio can be used in the examples.

Equation (2) gives the probability that any random point along the width of the rectangle that is selected as the entry point by a single aircraft will fall within the diameter of a single dust cloud. This is also the probability that an aircraft flying straight across the rectangle will cross the rectangle width in the interval D, and therefore fly through a dust cloud. For a given rectangle and cloud size, let the probability in Equation (2) be p. Then the probability that the aircraft will not fly through the cloud is given as (1 – p). When an aircraft flies through the rectangle, only two outcomes are possible: either the aircraft flies through a cloud, or it doesn't. This is a classical statistical problem that is described by the binomial distribution in Equation (3). [5:123]

$$b(x; n, p) = \binom{n}{x} p^x (1 - p)^{n-x} \qquad for\ \ 0 \le x \le n, \qquad 0\ otherwise \qquad (3)$$

Where:
p is the probability of flying through one randomly placed cloud, as calculated by Equation (2)
n is the total number of clouds in the rectangular area
x is the number of clouds that the aircraft will fly through, and
b(x;n,p) is the probability that an aircraft will fly through exactly x clouds, given n total clouds, and a probability p that the aircraft would fly through a single cloud in the area.

The combination of n and x in Equation (3) is defined as:

$$\binom{n}{x} = \frac{n!}{x!\,(n - x)!} \qquad (3.1)$$

Very often the analyst will be interested in the cumulative probability distribution of the binomial distribution, or:

$$P(X \leq x) = B(x; n, p) = \sum_{y=0}^{x} b(y; n, p) \qquad 0 \leq x \leq n \qquad (4)$$

The calculated value in Equation (4) is the cumulative probability that the aircraft will fly through 0, 1, 2… up to x clouds. Sometimes the analyst wants to know the probability that the aircraft will fly through at least one cloud. This can be calculated from:

$$P(1 \leq X \leq n) = \sum_{y=1}^{n} b(y; n, p) = 1 - b(0; n, p) = 1 - (1 - p)^n \qquad (5)$$

Examples for Clouds All the Same Size

For the first example, let's consider an area which is 10 km wide, with 4 clouds with a diameter of 1 km, which are randomly placed in the area. From Equation (2) the probability that we will fly through one randomly placed cloud is:

$$p = \frac{D}{W} = \frac{1}{10} = 0.10 \qquad (6)$$

The calculated p in Equation (6) is the p that we will use in equations (3) through (5). Table 1 below shows the probability of flying through 0, 1, 2, 3, or 4 clouds for this example.

Table 1—Probability table for 4 clouds in an area

x	b(x; n, p) = b(x; 4, 0.10)	B(x; 4, 0.10)
0	0.6561	0.6561
1	0.2916	0.9477
2	0.0486	0.9963
3	0.0036	0.9999
4	0.0001	1.0000

The probability that an aircraft will fly through one or more clouds is, from Equation (5):

$$1 - (1 - 0.10)^4 = 0.3439$$

which is the sum of the individual probabilities in Table 1 for flying through 1, 2, 3, or 4 clouds.

For the second example, consider an area with a width of 1,000 km, and the area is filled with 500 randomly placed dust clouds with a diameter of 10 km. Figure 3 shows a plot of equations (3) and (4) for this example. The probability than an aircraft will fly through 5 clouds is about 0.18, and this is the highest probability for any number of clouds. For the cumulative curve, the

probability of an aircraft flying through 7 or fewer clouds is 0.87, and 10 or fewer clouds is 0.99. The red line in Figure 3 is the cumulative probability calculated from Equation (4).

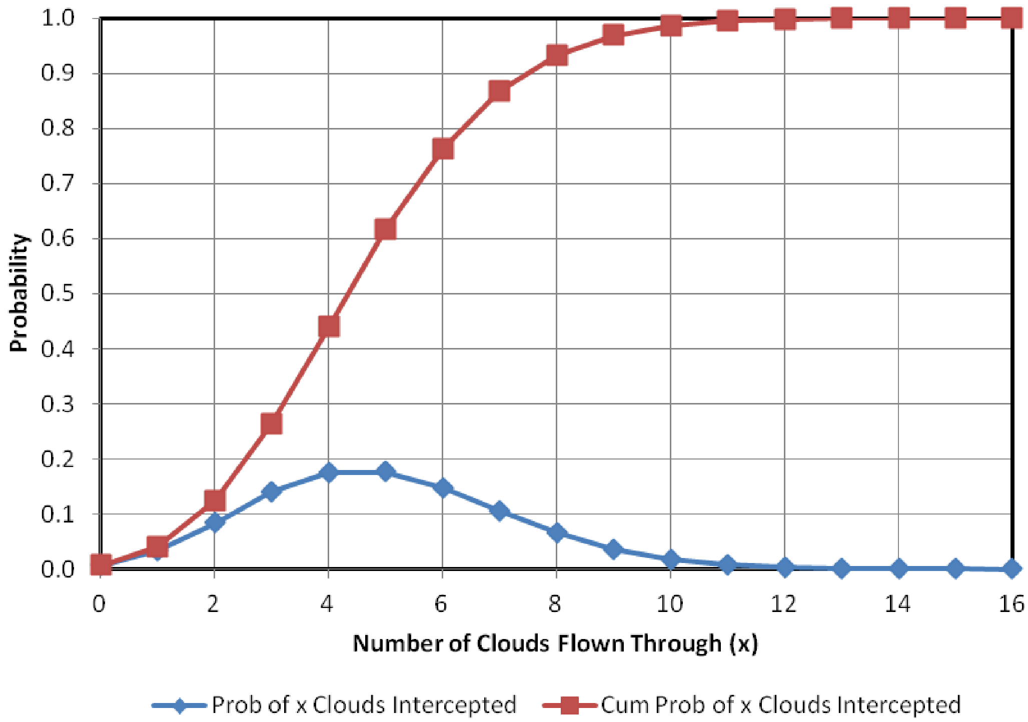

Figure 3-Probability of flying through dust clouds for example 2

Model Development for Different Size Clouds

The derivation above works so long as all of the dust clouds are the same size. The derivation will now be extended to account for clouds of different sizes. Suppose that there are two different cloud types with diameters of D_1 and D_2. Then from Equation (2), the probability of flying through one cloud of type 1 or 2 is:

$$p_1 = \frac{D_1}{W} \tag{7.1}$$

$$p_2 = \frac{D_2}{W} \tag{7.2}$$

If we imagine that all of the clouds of both types are randomly placed within the area, then the probability of flying through x clouds of type 1 is independent of the probability of flying through y clouds of type 2. If the clouds of type 1 and 2 are randomly placed within the area in Figure 2, then the probability of flying through x clouds of type 1, or the probability of flying through y clouds of type 2, can be calculated using Equation (3).

Let x and y be two independent variables with distribution functions from Equation (3).

$$\mathrm{P}(x) = b_1(x; n, p_1) = \binom{n}{x} p_1^x (1 - p_1)^{n-x} \tag{8.1}$$

$$\mathrm{P}(y) = b_2(y; m, p_2) = \binom{m}{y} p_2^y (1 - p_2)^{m-y} \tag{8.2}$$

Where:

p_1 is the probability of flying through one randomly placed cloud of type 1,
p_2 is the probability of flying through one randomly placed cloud of type 2,
n is the total number of clouds of type 1 in the rectangular area,
m is the total number of clouds of type 2 in the rectangular area,
x is the number of clouds of type 1 that the aircraft will fly through, and
y is the number of clouds of type 2 that the aircraft will fly through.

To get the probability of intersecting a given number of either type 1 or type 2 clouds in the sample space, the independent variables can be added.

$$z = x + y \tag{8.3}$$

Where x and y are from equations (8.1) and (8.2), and z is the total number of clouds of either type that will be flown through.

Given that the total number of type 1 clouds is n and the total number of type 2 clouds is m then Equation (8.4) can be used to calculate the probability that an aircraft will fly through any number z of randomly placed clouds in the rectangular area. The equation for the probability of intersecting any number of clouds is derived from the combination of equations (8.1) and (8.2), for each value of x and y that sum to the desired z. Since the variables x and y are discrete and independent in the sample space, the two binomial probabilities may be combined as shown in Equation (8.4)[6].

In order to find the probability of an aircraft flying through a particular number of clouds:

$$P(z) = \sum_{j=0}^{z} b_1(j) b_2(z - j) \quad 0 \le z \le m + n, \quad 0 \; otherwise \tag{8.4}$$

Where:
z is the total number of clouds of either type to be flown through,
$b_1(j) = b_1(j; n, p_1)$ from Equation (8.1), and
$b_2(z - j) = b_2(z - j; m, p_2)$ from Equation (8.2).

Example for 2 Different Size Clouds

This example calculates the probability of flying through a given number of clouds of two different types. Consider an area which is 10 km wide that contains 2 clouds of the first type, and

3 clouds of the second type. Let the diameter for the first type of cloud be 0.5 km, and let the diameter for the second type of cloud be 1 km. Table 2 shows the individual probabilities associated with each cloud type distribution using Equations (8.1) and (8.2). Since there are 5 total clouds of both types in the rectangular area, we need the probability of flying through up to 5 clouds of each type to be able to calculate all of the terms in Equation (8.4) when z = 5. Since there are only 2 clouds of the first type, the probability of flying through 3, 4 or 5 clouds of the first type is 0. Equation (8.4) uses these values to calculate the probability of flying through up to 5 clouds of either type. Table 3 shows the probabilities of flying through a given number of clouds of either type using Equation (8.4). In these and the following tables, when the probability is shown as 0, that value is determined by the definition of the probability distribution, such as for $x > n$ in Equation (3). When the 0 probability is shown with more than one significant figure, such as 0.0000, then the probability was calculated by an equation such as Equation (8.4).

Table 2—Probability table for flying through a given number of clouds of each type

	Cloud Type 1	Cloud Type 2
x	$b_1(x;2,0.05)$	$b_2(x;3,0.1)$
0	0.9025	0.7290
1	0.0950	0.2430
2	0.0025	0.0270
3	0	0.0010
4	0	0
5	0	0

An example calculation for the P(z = 3) row in Table 3 is given in equations (9.1) through (9.4) using Equation (8.4), and the values from Table 2.

$$P(z=3) = \sum_{j=0}^{3} b_1(j) b_2(3-j) \tag{9.1}$$

$$P(3) = b_1(0)b_2(3) + b_1(1)b_2(2) + b_1(2)b_2(1) + b_1(3)b_2(0) \tag{9.2}$$

$$P(3) = 0.9025 \cdot 0.0010 + 0.0950 \cdot 0.0270 + 0.0025 \cdot 0.0240 + 0 \cdot 0.7290 \tag{9.3}$$

$$P(3) = 0.0041 \tag{9.4}$$

Table 3—Probability table for flying through a given number of clouds of either type

Number of Clouds	Probability P(z)	Formula
z = 0	0.6579	$b_1(0) * b_2(0)$
z = 1	0.2886	$\sum_{j=0}^{1} b_1(j) * b_2(1-j)$
z = 2	0.0493	$\sum_{j=0}^{2} b_1(j) * b_2(2-j)$
z = 3	0.0041	$\sum_{j=0}^{3} b_1(j) * b_2(3-j)$
z = 4	0.0002	$\sum_{j=0}^{4} b_1(j) * b_2(4-j)$
z = 5	0.0000	$\sum_{j=0}^{5} b_1(j) * b_2(5-j)$

In Equation (9.2), the first term in the summation is the probability that an aircraft will fly through 0 clouds of type 1, and 3 clouds of type 2. The second term is the probability that the aircraft will fly through 1 cloud of type 1, and 2 clouds of type 2. The third term is the probability that the aircraft will fly through 2 clouds of type 1 and 1 cloud of type 2. The last term is the probability that the aircraft will fly through 3 clouds of type 1, and 0 clouds of type 2. The probability that the aircraft will fly through three clouds of either type is the sum of the four terms. As shown in Table 3 and Equation (9.4), the probability of flying through exactly 3 clouds is 0.0041. A graph of the combined probability distribution using both types of clouds is shown in Figure 4.

Using data from Table 3, the probability of flying through 1 or more clouds of either type is calculated in Equation (10).

$$P(1 \leq z \leq 5) = 1 - P(z = 0) = 1 - 0.6579 = 0.3421 \qquad (10)$$

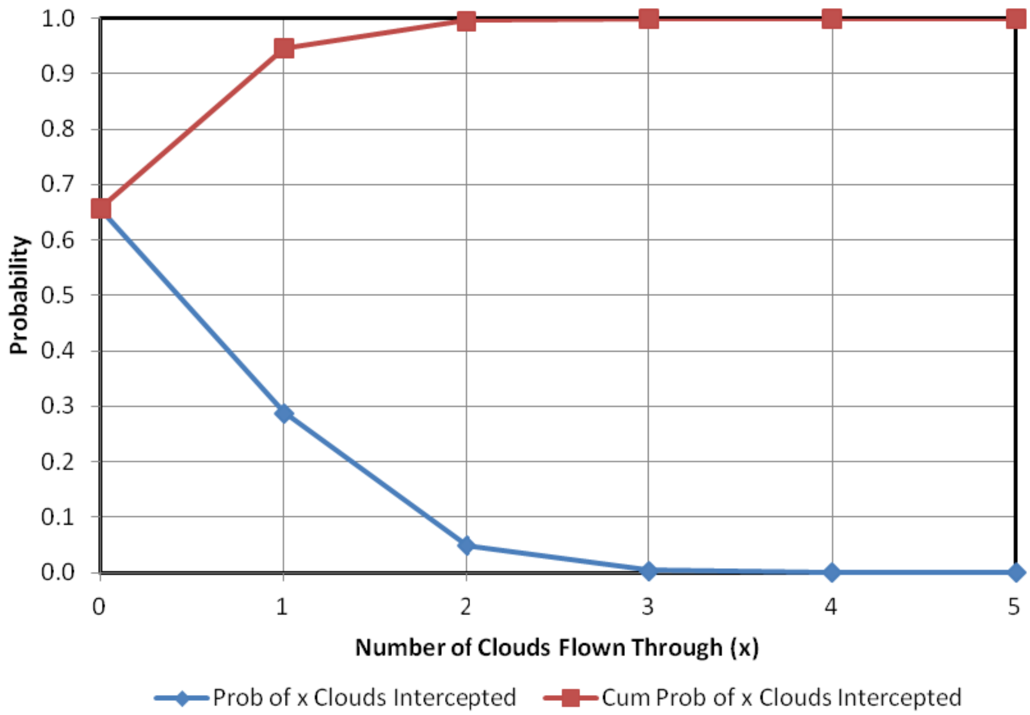

Figure 4-Probability of flying through dust clouds with 2 different types

Example for 4 Different Size Clouds

The ability to combine more than two random variables is merely an extension of equations (8.3) and (8.4). Assume we have the same 10 km wide area with 4 different cloud types. Further assume that the first two cloud types are the same as used in the previous example. There are 2 clouds of type 1, and they have a diameter of 0.5 km. There are 3 clouds of type 2 with a diameter of 1 km. For the type 3 clouds, assume that there are 2 clouds with a diameter of 0.2 km. For the type 4 clouds, assume that there are 6 clouds with a diameter of 2 km.

Table 4 shows the probability table for each of the four cloud types. Since there are 13 total clouds in the area, the table should extend down to an $x = 13$ row. These additional rows were omitted to save space, since all of the probabilities for $7 \leq x \leq 13$ are 0 in Table 4.

Table 4—Probability table for 4 cloud types in an area

	Cloud Type 1	Cloud Type 2	Cloud Type 3	Cloud Type 4
	Variable(x)	Variable(y)	Variable(r)	Variable(s)
x	b_1(x;2,0.05)	b_2(x;3,0.1)	b_3(x;2,0.02)	b_4(x;6,0.2)
0	0.9025	0.7290	0.9604	0.2621
1	0.0950	0.2430	0.0392	0.3932
2	0.0025	0.0270	0.0004	0.2458
3	0	0.0010	0	0.0819
4	0	0	0	0.0154
5	0	0	0	0.0015
6	0	0	0	0.0000
7	0	0	0	0

To incorporate all four of the cloud types we begin with Equation (8.3). The more cloud types that are added, the more iterations of Equation (8.4) that are needed. This is shown in equations (11.1), (11.2), and (11.3). The iterations are differentiated by a subscript for random variable *z*.

$$z_1 = x + y \qquad (11.1)$$

$$z_2 = z_1 + r \qquad (11.2)$$

$$z_3 = z_2 + s \qquad (11.3)$$

Where:
z_1 is the number of clouds flown through for the first iteration of Equation (11.1)
z_2 is the number of clouds flown through for the second iteration using z_1 and r.
z_3 is the number of clouds flown through for the third and final iteration using z_2 and s.
x and y represent the number of type 1 and 2 clouds flown through.
r and s represent the number of type 3 and 4 clouds flown through.

Table 5 shows the probabilities after the first iteration using Equation (11.1). The probabilities in the second column labeled “Cloud Type 1 & 2” are the same as the second column labeled “Probability P(z)” in Table 3. The second column gives the probabilities of flying through a given number of either type 1 or 2 clouds. This is the same as the results for the two different cloud types used in the previous example.

Table 5—Probability table for 4 cloud types in an area(1st iteration)

	Cloud Types 1 & 2	Cloud Type 3	Cloud Type 4
	Variable(z_1)	Variable(r)	Variable(s)
x	$b_{(1\&2)}$(x;5)	b_3(x;2,0.02)	b_4(x;6,0.2)
0	0.6579	0.9604	0.2621
1	0.2886	0.0392	0.3932
2	0.0493	0.0004	0.2458
3	0.0041	0	0.0819
4	0.0002	0	0.0154
5	0.0000	0	0.0015
6	0	0	0.0000
7	0	0	0

Table 6 shows the probability table after the second iteration using Equation (11.2). The second column labeled "Cloud Types 1 & 2 & 3" shows the probability of flying through a given number of clouds of either type 1, type 2, or type 3. The Equations (12) show the calculation for x = 2 in the second column of Table 6.

$$P(2) = \sum_{j=0}^{2} b_{(1\&2)}(j) b_3\,(2-j) \tag{12.1}$$

$$P(2) = 0.6579 \cdot 0.0004 + 0.2886 \cdot 0.0392 + 0.0493 \cdot 0.9604 \tag{12.2}$$

$$P(2) = 0.0589 \tag{12.3}$$

Table 6—Probability table for 4 cloud types in an area(2nd iteration)

	Cloud Types 1 & 2 & 3	Cloud Type 4
	Variable(z_2)	Variable(s)
x	$b_{(1\&2\&3)}(x;7)$	$b_4(x;6,0.2)$
0	0.6319	0.2621
1	0.3029	0.3932
2	0.0589	0.2458
3	0.0060	0.0819
4	0.0003	0.0154
5	0.0000	0.0015
6	0.0000	0.0000
7	0.0000	0

Figure 5 shows the probability curves for the second column of Table 6. There are 7 total clouds of types 1, 2, and 3.

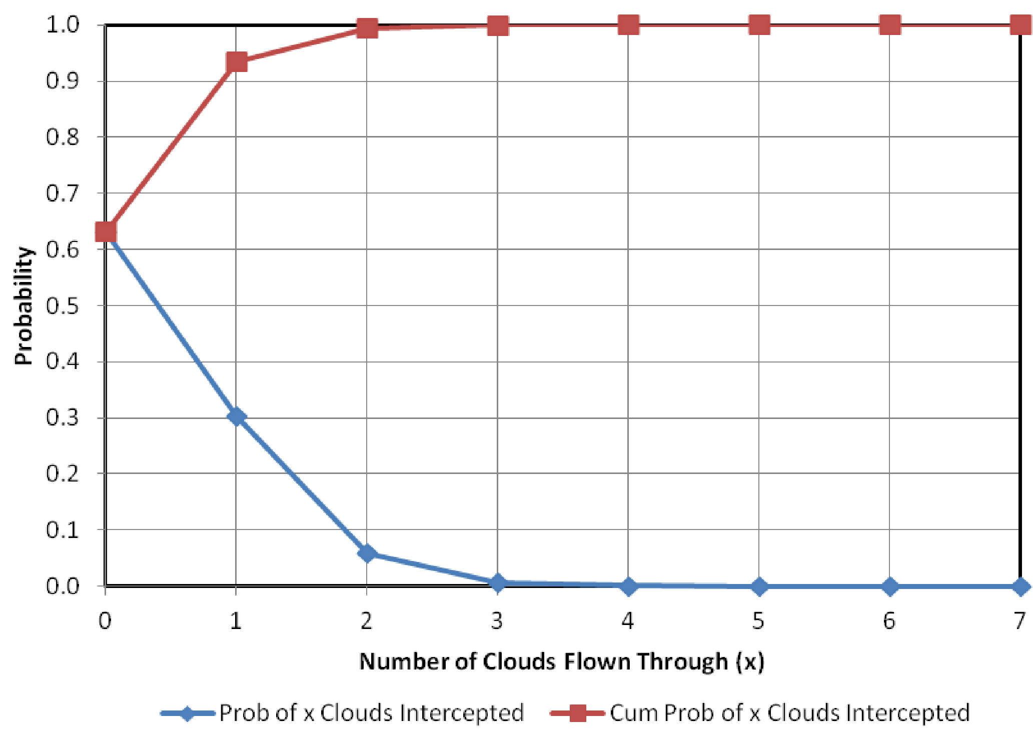

Figure 5-Probability of flying through dust clouds with 3 different sizes

Iterating again using Equation (11.3) gives the probabilities shown in the second column of Table 7. There are 13 total clouds of types 1, 2, 3, and 4. The probabilities in the second column are the probability of flying through x clouds of type 1, type 2, type 3, or type 4. Figure 6 shows the probability plot for flying through up to 13 clouds of the 4 different types.

Table 7—Probability table for 4 cloud types in an area(3rd and final iteration)

	Cloud Types 1 & 2 & 3 & 4
	Variable(z_3)
x	$b_{(1\&2\&3\&4)}(x;13)$
0	0.1656
1	0.3279
2	0.2898
3	0.1509
4	0.0514
5	0.0120
6	0.0020
7	0.0002
8	0.0000
9	0.0000
10	0.0000
11	0.0000
12	0.0000
13	0.0000

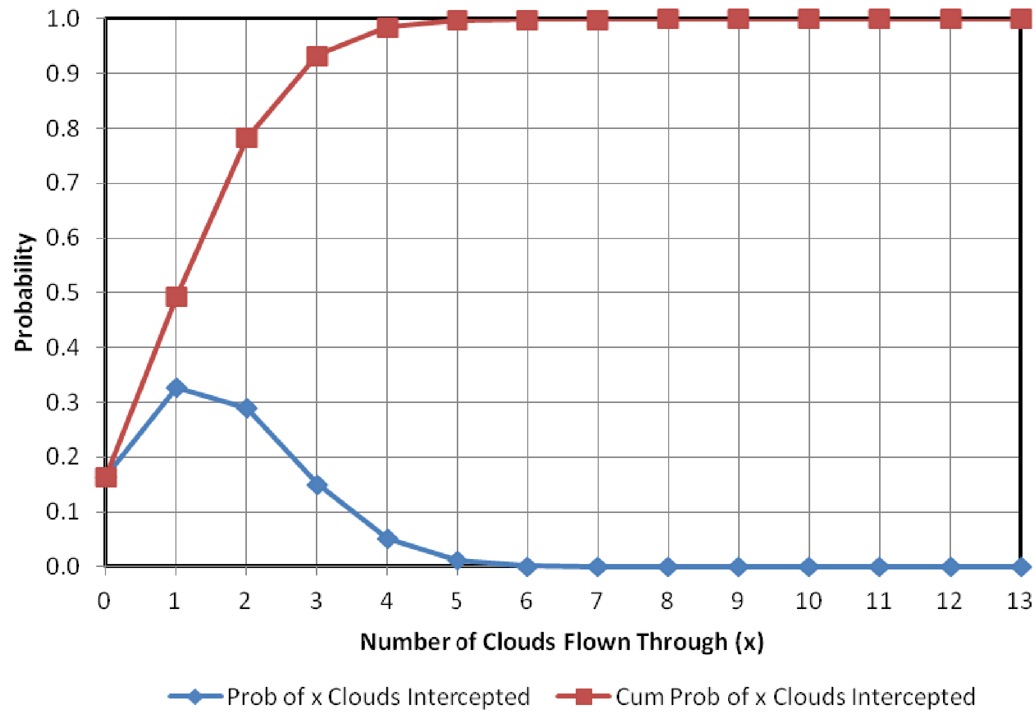

Figure 6-Probability of flying through dust clouds with 4 different sizes

Summary

This paper shows that there is a simple analytic method for calculating the probability of flying through a given number of nuclear dust clouds. Given the size of an area, the number of dust clouds, the size of the dust clouds, and assuming that the clouds are randomly placed within the area, an analyst can calculate the probability and cumulative probability of an aircraft flying through any specific number of dust clouds.

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14. ABSTRACT
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